

$F(4)$ Supergravity and 5D Superconformal Field Theories¹

Riccardo D'Auria¹, Sergio Ferrara² and Silvia Vaulá³

¹ Dipartimento di Fisica, Politecnico di Torino,
Corso Duca degli Abruzzi 24, I-10129 Torino

and Istituto Nazionale di Fisica Nucleare (INFN) - Sezione di Torino, Italy

² CERN Theoretical Division, CH 1211 Geneva 23, Switzerland

and Istituto Nazionale di Fisica Nucleare, Laboratori Nazionali di Frascati, Italy

³ Dipartimento di Fisica Teorica, Università di Torino, via P. Giuria 1, I-10125 Torino

Proceedings of the Gursey Memorial Conference II, "M-theory and Dualities", June 19-23 2000, Istanbul, Turkey.

1 Introduction

We report on a recent investigation [1] in which $F(4)$ supergravity [2], the gauge theory of the exceptional six-dimensional Anti-de Sitter superalgebra [3, 4], is coupled to an arbitrary number of vector multiplets whose scalar components parametrize the quaternionic manifold $SO(4, n)/SO(4) \times SO(n)$. By gauging the compact subgroup $SU(2)_d \otimes \mathcal{G}$, where $SU(2)_d$ is the diagonal subgroup of $SO(4) \simeq SU(2)_L \otimes SU(2)_R$ (the R -symmetry group of six-dimensional Poincaré supergravity) and \mathcal{G} is a compact group such that $\dim \mathcal{G} = n$, we obtain the scalar potential which, besides the gauge coupling constants, also depends in non trivial way on the parameter m associated to a massive 2-form $B_{\mu\nu}$ of the gravitational multiplet. The potential admits an AdS background for $g = 3m$, as the pure $F(4)$ -supergravity. We compute the scalar squared masses (which are all negative) which

¹The work of R. D'Auria and S. Vaulá has been supported by EEC under TMR contract ERBFMRX-CT96-0045, the work of S. Ferrara has been supported by the EEC TMR programme ERBFMRX-CT96-0045 (Laboratori Nazionali di Frascati, INFN) and by DOE grant DE-FG03-91ER40662, Task C.

are found to match the results dictated by AdS_6/CFT_5 correspondence [5, 6] from the conformal dimensions of boundary operators. The boundary $F(4)$ superconformal fields are realized in terms of a singleton superfield (hypermultiplet) in harmonic superspace with flag manifold $SU(2)/U(1) = S^2$. Finally we analyze the spectrum of short representations in terms of superconformal primaries and predict general features of the K-K spectrum of massive type IIA supergravity compactified on warped $AdS_6 \otimes S^4$.

2 A geometrical setting

In this section we set up a suitable framework for the discussion of the matter coupled $F(4)$ supergravity theory and its gauging. This will allow us to set up the formalism for the matter coupling in the next section. Actually we will just give the essential definitions of the Bianchi identities approach in superspace, while all the relevant results, specifically the supersymmetry transformation laws of the fields, will be given in the ordinary space-time formalism.

First of all it is useful to discuss the main results of ref. [2] by a careful study in superspace of the Poincaré and AdS supersymmetric vacua. Let us recall the content of $D = 6$, $N = (1, 1)$ supergravity multiplet:

$$(V_\mu^a, A_\mu^\alpha, B_{\mu\nu}, \psi_\mu^A, \psi_\mu^{\dot{A}}, \chi^A, \chi^{\dot{A}}, e^\sigma) \quad (2.1)$$

where V_μ^a is the six dimensional vielbein, $\psi_\mu^A, \psi_\mu^{\dot{A}}$ are left-handed and right-handed four-component gravitino fields respectively, A and \dot{A} transforming under the two factors of the R -symmetry group $O(4) \simeq SU(2)_L \otimes SU(2)_R$, $B_{\mu\nu}$ is a 2-form, A_μ^α ($\alpha = 0, 1, 2, 3$), are vector fields, $\chi^A, \chi^{\dot{A}}$ are left-handed and right-handed spin $\frac{1}{2}$ four components dilatinos, and e^σ denotes the dilaton.

Our notations are as follows: $a, b, \dots = 0, 1, 2, 3, 4, 5$ are Lorentz flat indices in $D = 6$, $\mu, \nu, \dots = 0, 1, 2, 3, 4, 5$ are the corresponding world indices, $A, \dot{A} = 1, 2$. Moreover our metric is $(+, -, -, -, -, -)$.

We recall that the description of the spinors of the multiplet in terms of left-handed and right-handed projection holds only in a Poincaré background, while in an AdS background the chiral projection cannot be defined and we are bounded to use 8-dimensional pseudo-Majorana spinors. In this case the R -symmetry group reduces to the $SU(2)$ subgroup of $SU(2)_L \otimes SU(2)_R$, the R -symmetry group of the chiral spinors. For our purposes, it is convenient to use from the very beginning 8-dimensional pseudo-Majorana spinors even in a Poincaré framework, since we are going to discuss in a unique setting both Poincaré and AdS vacua.

The pseudo-Majorana condition on the gravitino 1-forms is as follows:

$$(\psi_A)^\dagger \gamma^0 = \overline{(\psi_A)} = \epsilon^{AB} \psi_B^t \quad (2.2)$$

where we have chosen the charge conjugation matrix in six dimensions as the identity matrix (an analogous definition six dimensions as the identity matrix (an analogous definition holds for the dilatino fields). We use eight dimensional antisymmetric gamma matrices, with $\gamma^7 = i\gamma^0\gamma^1\gamma^2\gamma^3\gamma^4\gamma^5$, which implies $\gamma_7^T = -\gamma_7$ and $(\gamma_7)^2 = -1$. The indices $A, B, \dots = 1, 2$, of the spinor fields ψ_A, χ_A transform in the fundamental of the diagonal

subgroup $SU(2)$ of $SU(2)_L \otimes SU(2)_R$. For a generic $SU(2)$ tensor T , raising and lowering of indices are defined by

$$T^{\dots A \dots} = \epsilon^{AB} T^{\dots B \dots} \quad (2.3)$$

$$T_{\dots A \dots} = T_{\dots B \dots} \epsilon_{BA} \quad (2.4)$$

To study the supersymmetric vacua let us write down the Maurer-Cartan Equations (M.C.E.) dual to the $F(4)$ Superalgebra (anti)commutators:

$$\mathcal{D}V^a - \frac{i}{2} \bar{\psi}_A \gamma_a \psi^A = 0 \quad (2.5)$$

$$\mathcal{R}^{ab} + 4m^2 V^a V^b + m \bar{\psi}_A \gamma_{ab} \psi^A = 0 \quad (2.6)$$

$$dA^r + \frac{1}{2} g \epsilon^{rst} A_s A_t - i \bar{\psi}_A \psi_B \sigma^{rAB} = 0 \quad (2.7)$$

$$D\psi_A - im\gamma_a \psi_A V^a = 0 \quad (2.8)$$

Here $V^a, \omega^{ab}, \psi_A, A^r, (r = 1, 2, 3)$, are superfield 1-forms dual to the $F(4)$ supergenerators which at $\theta = 0$ have as dx^μ components

$$V_\mu^a = \delta_\mu^a, \quad \psi_{A\mu} = A_\mu^r = 0, \quad \omega_\mu^{ab} = \text{pure gauge}. \quad (2.9)$$

Furthermore $\mathcal{R}^{ab} \equiv d\omega^{ab} - \omega^{ac} \wedge \omega_c^b$, \mathcal{D} is the Lorentz covariant derivative, D is the $SO(1, 5) \otimes SU(2)$ covariant derivative, which on spinors acts as follows:

$$D\psi_A \equiv d\psi_A - \frac{1}{4} \gamma_{ab} \omega^{ab} \psi_A - \frac{i}{2} \sigma_{AB}^r A_r \psi^B \quad (2.10)$$

Note that $\sigma^{rAB} = \epsilon^{BC} \sigma_C^r$, where σ_B^r ($r = 1, 2, 3$) denote the usual Pauli matrices, are symmetric in A, B .

Let us point out that the $F(4)$ superalgebra, despite the presence of two different physical parameters, the $SU(2)$ gauge coupling constant g and the inverse AdS radius m , really depends on just one parameter since the closure under d -differentiation of eq. (2.8) (equivalent to the implementation of Jacobi identities on the generators), implies $g = 3m$; to recover this result one has to use the following Fierz identity involving 3- ψ_A 's 1-forms:

$$\frac{1}{4} \gamma_{ab} \psi_A \bar{\psi}_B \gamma^{ab} \psi_C \epsilon^{AC} - \frac{1}{2} \gamma_a \psi_A \bar{\psi}_B \gamma^a \psi_C \epsilon^{AC} + 3 \psi_C \bar{\psi}_B \psi_A \epsilon^{BC} = 0 \quad (2.11)$$

The $F(4)$ superalgebra described by equations (2.5) -(2.8) fails to describe the physical vacuum because of the absence of the superfields 2-form B and 1-form A^0 whose space-time restriction coincides with the physical fields $B_{\mu\nu}$ and A_μ^0 appearing in the supergravity multiplet. The recipe to have all the fields in a single algebra is well known and consists in considering the Free Differential Algebra (F.D.A.) [13] obtained from the $F(4)$ M.C.E.'s by adding two more equations for the 2-form B and for the 1-form A^0 (the 0-form fields χ_A and σ do not appear in the algebra since they are set equal to zero in the vacuum). It turns out that to have a consistent F.D.A. involving B and A^0 one has to add to the $F(4)$

M.C.E.'s two more equations involving dA^0 and dB ; in this way one obtains an extension of the M.C.E's to the following F.D.A:

$$\mathcal{D}V^a - \frac{i}{2} \bar{\psi}_A \gamma_a \psi^A = 0 \quad (2.12)$$

$$\mathcal{R}^{ab} + 4m^2 V^a V^b + m \bar{\psi}_A \gamma_{ab} \psi^A = 0 \quad (2.13)$$

$$dA^r + \frac{1}{2} g \epsilon^{rst} A_s A_t - i \bar{\psi}_A \psi_B \sigma^{rAB} = 0 \quad (2.14)$$

$$dA^0 - mB - i \bar{\psi}_A \gamma_7 \psi^A = 0 \quad (2.15)$$

$$dB + 2 \bar{\psi}_A \gamma_7 \gamma_a \psi^A V^a = 0 \quad (2.16)$$

$$D\psi_A - im\gamma_a \psi_A V^a = 0 \quad (2.17)$$

Equations (2.15) and (2.16) were obtained by imposing that they satisfy the d -closure together with equations (2.12). Actually the closure of (2.16) relies on the 4- ψ_A 's Fierz identity

$$\bar{\psi}_A \gamma_7 \gamma_a \psi_B \epsilon^{AB} \bar{\psi}_C \gamma^a \psi_D \epsilon^{CD} = 0 \quad (2.18)$$

The interesting feature of the F.D.A (2.12)-(2.17) is the appearance of the combination $dA^0 - mB$ in (2.15). That means that the dynamical theory obtained by gauging the F.D.A. out of the vacuum will contain the fields A_μ^0 and $B_{\mu\nu}$ always in the single combination $\partial_{[\mu} A_{\nu]}^0 - mB_{\mu\nu}$. At the dynamical level this implies, as noted by Romans [2], an Higgs phenomenon where the 2-form B "eats" the 1-form A^0 and acquires a non vanishing mass m ².

In summary, we have shown that two of the main results of [2], namely the existence of an AdS supersymmetric background only for $g = 3m$ and the Higgs-type mechanism by which the field $B_{\mu\nu}$ becomes massive acquiring longitudinal degrees of freedom in terms of the the vector A_μ^0 , are a simple consequence of the algebraic structure of the F.D.A. associated to the $F(4)$ supergroup written in terms of the vacuum-superfields.

It is interesting to see what happens if one or both the parameters g and m are zero. Setting $m = g = 0$, one reduces the $F(4)$ Superalgebra to the $D = 6$ $N = (1, 1)$ superalgebra existing only in a Super Poincaré background; in this case the four- vector $A^\alpha \equiv (A^0, A^r)$ transforms in the fundamental of the R -symmetry group $SO(4)$ while the pseudo-Majorana spinors ψ_A, χ_A can be decomposed in two chiral spinors in such a way that all the resulting F.D.A. is invariant under $SO(4)$.

Furthermore it is easy to see that no F.D.A exists if either $m = 0$, $g \neq 0$ or $m \neq 0$, $g = 0$, since the corresponding equations in the F.D.A. do not close anymore under d -differentiation. In other words the gauging of $SU(2)$, $g \neq 0$ must be necessarily accompanied by the presence of the parameter m which, as we have seen, makes the closure of the supersymmetric algebra consistent for $g = 3m$.

In $D = 6$, $N = 4$ Supergravity, the only kind of matter is given by vector multiplets, namely

$$(A_\mu, \lambda_A, \phi^\alpha)^I \quad (2.19)$$

where $\alpha = 0, 1, 2, 3$ and the index I labels an arbitrary number n of such multiplets. As it is well known the $4n$ scalars parametrize the coset manifold $SO(4, n)/SO(4) \times SO(n)$.

²An analogous phenomenon takes place also in $D = 5$; see[14].

Taking into account that the pure supergravity has a non compact duality group $O(1, 1)$ parametrized by e^σ , the duality group of the matter coupled theory is

$$G/H = \frac{SO(4, n)}{SO(4) \times SO(n)} \times O(1, 1) \quad (2.20)$$

To perform the matter coupling we follow the geometrical procedure of introducing the coset representative L^Λ_Σ of the matter coset manifold, where $\Lambda, \Sigma, \dots = 0, \dots, 3 + n$; decomposing the $O(4, n)$ indices with respect to $H = SO(4) \times O(n)$ we have:

$$L^\Lambda_\Sigma = (L^\Lambda_\alpha, L^\Lambda_I) \quad (2.21)$$

where $\alpha = 0, 1, 2, 3$, $I = 4, \dots, 3 + n$. Furthermore, since we are going to gauge the $SU(2)$ diagonal subgroup of $O(4)$ as in pure Supergravity, we will also decompose L^Λ_α as

$$L^\Lambda_\alpha = (L^\Lambda_0, L^\Lambda_r) \quad (2.22)$$

The $4 + n$ gravitational and matter vectors will now transform in the fundamental of $SO(4, n)$ so that the superspace vector curvatures will be now labeled by the index Λ : $F^\Lambda \equiv (F^0, F^r, F^I)$. Furthermore the covariant derivatives acting on the spinor fields will now contain also the composite connections of the $SO(4, n)$ duality group. Let us introduce the left-invariant 1-form of $SO(4, n)$

$$\Omega^\Lambda_\Sigma = (L^\Lambda_\Pi)^{-1} dL^\Pi_\Sigma \quad (2.23)$$

satisfying the Maurer-Cartan equation

$$d\Omega^\Lambda_\Sigma + \Omega^\Lambda_\Pi \wedge \Omega^\Pi_\Sigma = 0 \quad (2.24)$$

By appropriate decomposition of the indices, we find:

$$R^r_s = -P^r_I \wedge P^I_s \quad (2.25)$$

$$R^r_0 = -P^r_I \wedge P^I_0 \quad (2.26)$$

$$R^I_J = -P^I_r \wedge P^r_J - P^I_0 \wedge P^0_J \quad (2.27)$$

$$\nabla P^I_r = 0 \quad (2.28)$$

$$\nabla P^I_0 = 0 \quad (2.29)$$

where

$$R^{rs} \equiv d\Omega^r_s + \Omega^r_t \wedge \Omega^t_s + \Omega^r_0 \wedge \Omega^0_s \quad (2.30)$$

$$R^{r0} \equiv d\Omega^r_0 + \Omega^r_t \wedge \Omega^t_0 \quad (2.31)$$

$$R^{IJ} \equiv d\Omega^I_J + \Omega^I_K \wedge \Omega^K_J \quad (2.32)$$

and we have set

$$P^I_\alpha = \begin{cases} P^I_0 \equiv \Omega^I_0 \\ P^I_r \equiv \Omega^I_r \end{cases}$$

Note that P^I_0 , P^I_r are the vielbeins of the coset, while $(\Omega^{rs}, \Omega^{r0})$, (R^{rs}, R^{r0}) are respectively the connections and the curvatures of $SO(4)$ decomposed with respect to the

diagonal subgroup $SU(2) \subset SO(4)$.

In terms of the previous definitions, the ungauged superspace curvatures of the matter coupled theory, (with $m = 0$) are now given by:

$$T^A = \mathcal{D}V^a - \frac{i}{2} \bar{\psi}_A \gamma_a \psi^A V^a = 0 \quad (2.33)$$

$$R^{ab} = \mathcal{R}^{ab} \quad (2.34)$$

$$H = dB + 2e^{-2\sigma} \bar{\psi}_A \gamma_7 \gamma_a \psi^A V^a \quad (2.35)$$

$$F^\Lambda = \mathcal{F}^\Lambda - ie^\sigma L_0^\Lambda \epsilon^{AB} \bar{\psi}_A \gamma_7 \psi_B - ie^\sigma L_r^\Lambda \sigma^{rAB} \bar{\psi}_A \psi_B \quad (2.36)$$

$$\rho_A = \mathcal{D}\psi_A - \frac{i}{2} \sigma_{rAB} \left(-\frac{1}{2} \epsilon^{rst} \Omega_{st} - i\gamma_7 \Omega_{r0} \right) \psi^B \quad (2.37)$$

$$D\chi_A = \mathcal{D}\chi_A - \frac{i}{2} \sigma_{rAB} \left(-\frac{1}{2} \epsilon^{rst} \Omega_{st} - i\gamma_7 \Omega_{r0} \right) \chi^B \quad (2.38)$$

$$R(\sigma) = d\sigma \quad (2.39)$$

$$\nabla \lambda_{IA} = \mathcal{D}\lambda_{IA} - \frac{i}{2} \sigma_{rAB} \left(-\frac{1}{2} \epsilon^{rst} \Omega_{st} - i\gamma_7 \Omega_{r0} \right) \lambda_I^B \quad (2.40)$$

$$R_0^I(\phi) \equiv P_0^I \quad (2.41)$$

$$R_r^I(\phi) \equiv P_r^I \quad (2.42)$$

where the last two equations define the "curvatures" of the matter scalar fields ϕ^i as the vielbein of the coset:

$$P_0^I \equiv P_{0i}^I d\phi^i \quad P_r^I \equiv P_{ri}^I d\phi^i \quad (2.43)$$

where i runs over the $4n$ values of the coset vielbein world-components.

As in the pure supergravity case one can now write down the superspace Bianchi identities for the matter coupled curvatures. The computation is rather long but straightforward. We limit ourselves to give the new transformation laws of all the physical fields when matter is present, as derived from the solutions of the Bianchi identities.

$$\delta V_\mu^a = -i\bar{\psi}_{A\mu} \gamma^a \varepsilon^A \quad (2.44)$$

$$\delta B_{\mu\nu} = 2e^{-2\sigma} \bar{\chi}_A \gamma_7 \gamma_{\mu\nu} \varepsilon^A - 4e^{-2\sigma} \bar{\varepsilon}_A \gamma_7 \gamma_{[\mu} \psi_{\nu]}^A \quad (2.45)$$

$$\begin{aligned} \delta A_\mu^\Lambda &= 2e^\sigma \bar{\varepsilon}^A \gamma_7 \gamma_\mu \chi^B L_0^\Lambda \epsilon_{AB} + 2e^\sigma \bar{\varepsilon}^A \gamma_\mu \chi^B L^{\Lambda r} \sigma_{rAB} - e^\sigma L_{\Lambda I} \bar{\varepsilon}^A \gamma_\mu \lambda^{IB} \epsilon_{AB} + \\ &+ 2ie^\sigma L_0^\Lambda \bar{\varepsilon}_A \gamma^7 \psi_B \epsilon^{AB} + 2ie^\sigma L^{\Lambda r} \sigma_r^{AB} \bar{\varepsilon}_A \psi_B \end{aligned} \quad (2.46)$$

$$\begin{aligned} \delta \psi_{A\mu} &= \mathcal{D}_\mu \varepsilon_A + \frac{1}{16} e^{-\sigma} [T_{[AB]\nu\lambda} \gamma_7 - T_{(AB)\nu\lambda}] (\gamma_\mu^{\nu\lambda} - 6\delta_\mu^\nu \gamma^\lambda) \varepsilon^B + \\ &+ \frac{i}{32} e^{2\sigma} H_{\nu\lambda\rho} \gamma_7 (\gamma_\mu^{\nu\lambda\rho} - 3\delta_\mu^\nu \gamma^{\lambda\rho}) \varepsilon_A + \frac{1}{2} \varepsilon_A \bar{\chi}^C \psi_{C\mu} + \\ &+ \frac{1}{2} \gamma_7 \varepsilon_A \bar{\chi}^C \gamma^7 \psi_{C\mu} - \gamma_\nu \varepsilon_A \bar{\chi}^C \gamma^\nu \psi_{C\mu} + \gamma_7 \gamma_\nu \varepsilon_A \bar{\chi}^C \gamma^7 \gamma^\nu \psi_{C\mu} + \\ &- \frac{1}{4} \gamma_\nu \lambda \varepsilon_A \bar{\chi}^C \gamma^{\nu\lambda} \psi_{C\mu} - \frac{1}{4} \gamma_7 \gamma_\nu \lambda \varepsilon_A \bar{\chi}^C \gamma^7 \gamma^{\nu\lambda} \psi_{C\mu} \end{aligned} \quad (2.47)$$

$$\delta \chi_A = \frac{i}{2} \gamma^\mu \partial_\mu \sigma \varepsilon_A + \frac{i}{16} e^{-\sigma} [T_{[AB]\mu\nu} \gamma_7 + T_{(AB)\mu\nu}] \gamma^{\mu\nu} \varepsilon^B + \frac{1}{32} e^{2\sigma} H_{\mu\nu\lambda} \gamma_7 \gamma^{\mu\nu\lambda} \varepsilon_A \quad (2.48)$$

$$\delta \sigma = \bar{\chi}_A \varepsilon^A \quad (2.49)$$

$$\delta \lambda^{IA} = -iP_{ri}^I \sigma^{rAB} \partial_\mu \phi^i \gamma^\mu \varepsilon_B + iP_{0i}^I \epsilon^{AB} \partial_\mu \phi^i \gamma^7 \gamma^\mu \varepsilon_B + \frac{i}{2} e^{-\sigma} T_{\mu\nu}^I \gamma^{\mu\nu} \varepsilon^A \quad (2.50)$$

$$P_{0i}^I \delta\phi^i = \frac{1}{2} \bar{\lambda}_A^I \gamma_7 \varepsilon^A \quad (2.51)$$

$$P_{ri}^I \delta\phi^i = \frac{1}{2} \bar{\lambda}_A^I \varepsilon_B \sigma_r^{ab} \quad (2.52)$$

where we have introduced the "dressed" vector field strengths

$$T_{[AB]\mu\nu} \equiv \epsilon_{AB} L_{0\Lambda}^{-1} F_{\mu\nu}^\Lambda \quad (2.53)$$

$$T_{(AB)\mu\nu} \equiv \sigma_{AB}^r L_{r\Lambda}^{-1} F_{\mu\nu}^\Lambda \quad (2.54)$$

$$T_{I\mu\nu} \equiv L_{I\Lambda}^{-1} F_{\mu\nu}^\Lambda \quad (2.55)$$

and we have omitted in the transformation laws of the fermions the three-fermions terms of the form $(\chi\chi\varepsilon)$, $(\lambda\lambda\varepsilon)$.

3 The gauging

The next problem we have to cope with is the gauging of the matter coupled theory and the determination of the scalar potential.

Let us first consider the ordinary gauging, with $m = 0$, which, as usual, will imply the presence of new terms proportional to the coupling constants in the supersymmetry transformation laws of the fermion fields.

Our aim is to gauge a compact subgroup of $O(4, n)$. Since in any case we may gauge only the diagonal subgroup $SU(2) \subset O(4) \subset H$, the maximal gauging is given by $SU(2) \otimes \mathcal{G}$ where \mathcal{G} is a n -dimensional subgroup of $O(n)$. According to a well known procedure, we modify the definition of the left invariant 1-form $L^{-1}dL$ by replacing the ordinary differential with the $SU(2) \otimes \mathcal{G}$ covariant differential as follows:

$$\nabla L^\Lambda{}_\Sigma = dL^\Lambda{}_\Sigma - f_{\Gamma\Pi}^\Lambda A^\Gamma L^\Pi{}_\Sigma \quad (3.1)$$

where $f_{\Pi\Gamma}^\Lambda$ are the structure constants of $SU(2)_d \otimes \mathcal{G}$. More explicitly, denoting with ϵ^{rst} and \mathcal{C}^{IJK} the structure constants of the two factors $SU(2)$ and \mathcal{G} , equation (3.1) splits as follows:

$$\nabla L^0{}_\Sigma = dL^0{}_\Sigma \quad (3.2)$$

$$\nabla L^r{}_\Sigma = dL^r{}_\Sigma - g \epsilon_t{}^r{}_s A^t L^s{}_\Sigma \quad (3.3)$$

$$\nabla L^I{}_\Sigma = dL^I{}_\Sigma - g' \mathcal{C}_K{}^I{}_J A^K L^J{}_\Sigma \quad (3.4)$$

Setting $\hat{\Omega} = L^{-1}\nabla L$, one easily obtains the gauged Maurer-Cartan equations:

$$d\hat{\Omega}^\Lambda{}_\Sigma + \hat{\Omega}^\Lambda{}_\Pi \wedge \hat{\Omega}^\Pi{}_\Sigma = (L^{-1}\mathcal{F}L)^\Lambda{}_\Sigma \quad (3.5)$$

where $\mathcal{F} \equiv \mathcal{F}^\Lambda T_\Lambda$, T_Λ being the generators of $SU(2) \otimes \mathcal{G}$.

After gauging, the same decomposition as in eqs. (2.25) -(2.29) now gives:

$$R^r{}_s = -P^r{}_I \wedge P^I{}_s + (L^{-1}\mathcal{F}L)^r{}_s \quad (3.6)$$

$$R^r{}_0 = -P^r{}_I \wedge P^I{}_0 + (L^{-1}\mathcal{F}L)^r{}_0 \quad (3.7)$$

$$R^I{}_J = -P^I{}_r \wedge P^r{}_J - P^I{}_0 \wedge P^0{}_J + (L^{-1}\mathcal{F}L)^I{}_J \quad (3.8)$$

$$\nabla P^I{}_r = (L^{-1}\mathcal{F}L)^I{}_r \quad (3.9)$$

$$\nabla P^I{}_0 = (L^{-1}\mathcal{F}L)^I{}_0 \quad (3.10)$$

Because of the presence of the gauged terms in the coset curvatures, the new Bianchi Identities are not satisfied by the old superspace curvatures but we need extra terms in the fermion field strengths parametrizations, that is, in space-time language, extra terms in the transformation laws of the fermion fields of eqs. (2.47), (2.48), (2.50), named “fermionic shifts”.

$$\delta\psi_{A\mu} = \delta\psi_{A\mu}^{(old)} + S_{AB}(g, g')\gamma_\mu\varepsilon^B \quad (3.11)$$

$$\delta\chi_A = \delta\chi_A^{(old)} + N_{AB}(g, g')\varepsilon^B \quad (3.12)$$

$$\delta\lambda_A^I = \delta\lambda_A^{I(old)} + M_{AB}^I(g, g')\varepsilon^B \quad (3.13)$$

Again, working out the Bianchi identities, one fixes the explicit form of the fermionic shifts which turn out to be

$$S_{AB}^{(g, g')} = \frac{i}{24}Ae^\sigma\epsilon_{AB} - \frac{i}{8}B_t\gamma^7\sigma_{AB}^t \quad (3.14)$$

$$N_{AB}^{(g, g')} = \frac{1}{24}Ae^\sigma\epsilon_{AB} + \frac{1}{8}B_t\gamma^7\sigma_{AB}^t \quad (3.15)$$

$$M_{AB}^{I(g, g')} = (-C_t^I + 2i\gamma^7 D_t^I)\sigma_{AB}^t \quad (3.16)$$

where

$$A = \epsilon^{rst}K_{rst} \quad (3.17)$$

$$B^i = \epsilon^{ijk}K_{jk0} \quad (3.18)$$

$$C_I^t = \epsilon^{trs}K_{rIs} \quad (3.19)$$

$$D_{It} = K_{0It} \quad (3.20)$$

and the threefold completely antisymmetric tensors K 's are the so called “boosted structure constants” given explicitly by:

$$K_{rst} = g\epsilon_{lmn}L^l{}_r(L^{-1})_s{}^m L^n{}_t + g'\mathcal{C}_{IJK}L^I{}_r(L^{-1})_s{}^J L^K{}_t \quad (3.21)$$

$$K_{rs0} = g\epsilon_{lmn}L^l{}_r(L^{-1})_s{}^m L^n{}_0 + g'\mathcal{C}_{IJK}L^I{}_r(L^{-1})_s{}^J L^K{}_0 \quad (3.22)$$

$$K_{rIt} = g\epsilon_{lmn}L^l{}_r(L^{-1})_I{}^m L^n{}_t + g'\mathcal{C}_{LJK}L^L{}_r(L^{-1})_I{}^J L^K{}_t \quad (3.23)$$

$$K_{0It} = g\epsilon_{lmn}L^l{}_0(L^{-1})_I{}^m L^n{}_t + g'\mathcal{C}_{LJK}L^L{}_0(L^{-1})_I{}^J L^K{}_t \quad (3.24)$$

Actually one easily see that the fermionic shifts (3.14) (3.15) reduce to the pure supergravity g dependent terms of equations (2.39) and (2.40) of reference [1]. (Note that, since $L^\Lambda_\Sigma \rightarrow \delta^\Lambda_\Sigma$ in absence of matter, the terms proportional to the Pauli σ matrices are simply absent in such a limit.)

At this point one could compute the scalar potential of the matter coupled theory, in terms of the fermionic shifts just determined, using the well known Ward identity of the scalar potential which can be derived from the Lagrangian. Since we are going to perform this derivation once we will introduce also m dependent terms in the fermionic shifts, we just quote, for the moment the expected result that the potential due only to g and g' dependent shifts doesn't admit a stable AdS background configuration. We are thus, led as in the pure supergravity case, to determine suitable m dependent terms that reduce to the m terms of eqs. (2.56) and (2.57) of reference [1] in absence of matter multiplets (one can see that the simple-minded ansatz of keeping exactly the same form for the

m dependent terms as in the pure Supergravity case is not consistent with the gauged superspace Bianchi identities).

It turns out that a consistent solution for the relevant m terms to be added to the fermionic shifts, implies the presence of the coset representatives; that is, the m -terms must also be "dressed" with matter scalar fields as it happens for the g and g' dependent terms. Explicitly, the Bianchi identities solution for the new fermionic shifts is:

$$S_{AB}^{(g,g',m)} = \frac{i}{24}[Ae^\sigma + 6me^{-3\sigma}(L^{-1})_{00}]\epsilon_{AB} - \frac{i}{8}[B_te^\sigma - 2me^{-3\sigma}(L^{-1})_{i0}]\gamma^7\sigma_{AB}^t \quad (3.25)$$

$$N_{AB}^{(g,g',m)} = \frac{1}{24}[Ae^\sigma - 18me^{-3\sigma}(L^{-1})_{00}]\epsilon_{AB} + \frac{1}{8}[B_te^\sigma + 6me^{-3\sigma}(L^{-1})_{i0}]\gamma^7\sigma_{AB}^t \quad (3.26)$$

$$M_{AB}^{I(g,g',m)} = (-C_t^I + 2i\gamma^7 D_t^I)e^\sigma\sigma_{AB}^t - 2me^{-3\sigma}(L^{-1})^I{}_0\epsilon_{AB} \quad (3.27)$$

4 The scalar potential

The simplest way to derive the scalar potential is to use the supersymmetry Ward identity which relates the scalar potential to the fermionic shifts in the transformation laws [16]. In order to retrieve such identity it is necessary to have the relevant terms of the Lagrangian of the gauged theory. These terms are actually the kinetic ones and the "mass" terms given in the following equation:

$$\begin{aligned} (detV)^{-1}\mathcal{L} = & -\frac{1}{4}\mathcal{R} - \frac{1}{8}e^{2\sigma}\mathcal{N}_{\Lambda\Sigma}\hat{\mathcal{F}}_{\mu\nu}^\Lambda\hat{\mathcal{F}}^{\Sigma\mu\nu} + \partial^\mu\sigma\partial_\mu\sigma - \frac{1}{4}(P_\mu^{I0}P_{I0}^\mu + P_\mu^{Ir}P_{Ir}^\mu) + \\ & -\frac{i}{2}\bar{\psi}_{A\mu}\gamma^{\mu\nu\rho}D_\nu\psi_\rho^A + \frac{i}{8}\bar{\lambda}_A^I\gamma^\mu D_\mu\lambda_I^A - 2i\bar{\chi}_A\gamma^\mu D_\mu\chi^A + 2i\bar{\psi}_\mu^A\gamma^{\mu\nu}\bar{S}_{AB}\psi_\nu^B + \\ & + 4i\bar{\psi}_\mu^A\gamma^\mu\bar{N}_{AB}\chi^B + \frac{i}{4}\bar{\psi}_\mu^A\gamma^\mu\bar{M}_{AB}^I\lambda_I^B + \mathcal{W}(\sigma\phi^i; g, g', m) + \dots \end{aligned} \quad (4.1)$$

where

$$\mathcal{N}_{\Lambda\Sigma} = L_\Lambda{}^0 L_{0\Sigma}^{-1} + L_\Lambda{}^i L_{i\Sigma}^{-1} - L_\Lambda{}^I L_{I\Sigma}^{-1} \quad (4.2)$$

is the vector kinetic matrix, $\hat{\mathcal{F}}_{\mu\nu}^\Lambda \equiv \mathcal{F}_{\mu\nu}^\Lambda - m\delta_0^\Lambda B_{\mu\nu}$ and \mathcal{W} is minus the scalar potential.

In equation (4.1) there appear "barred mass-matrices" \bar{S}_{AB} , \bar{N}_{AB} , \bar{M}_{AB}^I which are slightly different from the fermionic shifts defined in eqs. (3.25), (3.26), (3.27). Actually they are defined by:

$$\bar{S}_{AB} = -S_{BA}, \quad \bar{N}_{AB} = -N_{BA}, \quad \bar{M}_{AB}^I = M_{BA}^I \quad (4.3)$$

Definitions (4.3) stem from the fact that the shifts defined in eqs. (3.25), (3.26), (3.27) are matrices in the eight-dimensional spinor space, since they contain the γ_7 matrix; as will be seen in a moment, such definition is actually necessary in order to satisfy the supersymmetry Ward identity.

Indeed, let us perform the supersymmetry variation of (4.1), keeping only the terms proportional to g , g' or m , and to the current $\bar{\psi}_{A\mu}\gamma^\mu\epsilon^A$; we find the following Ward identity, :

$$\delta_A^C \mathcal{W} = 20\bar{S}^{AB}S_{BC} + 4\bar{N}^{AB}N_{BC} + \frac{1}{4}\bar{M}_{AB}^{AB}M_{BC}^I \quad (4.4)$$

However we note that, performing the supersymmetry variation, the gauge terms also give rise to extra terms proportional to the current $\bar{\psi}_{A\mu}\gamma^7\gamma^\mu\epsilon^A$, which have no counterpart in

the term containing the potential \mathcal{W} . Because of the definition of the barred mass matrices in eq. (4.3) it is easily seen that such " γ^7 -terms", arising from $\overline{S}^{AB}S_{BC}$ and $\overline{N}^{AB}N_{BC}$ cancel against each other.

As far as the term $\overline{M}_I^{AB}M_{BC}^I$ is concerned, the same mechanism of cancellation again applies to the terms proportional to $\overline{\psi}_{A\mu}\gamma_7\gamma^\mu\epsilon^A\sigma_C^{rA}$; there is, however, a residual dangerous term of the form

$$\delta_C^A\overline{\psi}_{A\mu}\gamma^\mu\gamma^7D_s^I C_I^s\epsilon^C \quad (4.5)$$

One can show that this term vanishes identically owing to the non trivial relation

$$D_t^I C_I^t = 0 \quad (4.6)$$

Equation (4.6) can be shown to hold using the pseudo-orthogonality relation $L^T\eta L = \eta$ among the coset representatives and the Jacobi identities $C_{I[JK}C_{L]MN} = 0$, $\epsilon_{r[st}\epsilon_{l]mn} = 0$. This is a non trivial check of our computation.

It now follows that the Ward identity eq. (4.4) is indeed satisfied since all the terms on the r.h.s., once the " γ^7 -terms" have been cancelled, are proportional to δ_A^C .

Using the expressions (3.25), (3.26), (3.27), (4.3) in equation (4.4), we obtain the explicit form of the scalar potential

$$\begin{aligned} \mathcal{W}(\phi) = 5 \{ & [\frac{1}{12}(Ae^\sigma + 6me^{-3\sigma}L_{00})]^2 + [\frac{1}{4}(e^\sigma B_i - 2me^{-3\sigma}L_{0i})]^2\} + \\ & - \{ [\frac{1}{12}(Ae^\sigma - 18me^{-3\sigma}L_{00})]^2 + [\frac{1}{4}(e^\sigma B_i + 6me^{-3\sigma}L_{0i})]^2\} + \\ & - \frac{1}{4}\{C_t^I C_{It} + 4D_t^I D_{It}\} e^{2\sigma} - m^2 e^{-6\sigma} L_{0I} L^{0I} \end{aligned} \quad (4.7)$$

Expanding the squares in equation (4.7) the potential \mathcal{W} can be alternatively written as follows:

$$\begin{aligned} \mathcal{W} = e^{2\sigma} [& \frac{1}{36}A^2 + \frac{1}{4}B^i B_i - \frac{1}{4}(C_t^I C_{It} + 4D_t^I D_{It})] - m^2 e^{-6\sigma} \mathcal{N}_{00} + \\ & + me^{-2\sigma} [\frac{2}{3}AL_{00} - 2B^i L_{0i}] \end{aligned} \quad (4.8)$$

where \mathcal{N}_{00} is the 00 component of the vector kinetic matrix defined in eq. (4.2).

We now show that, apart from other possible extrema not considered here, a stable supersymmetric extremum of the potential \mathcal{W} is found to be the same as in the case of pure supergravity, that is we get an AdS supersymmetric background only for $g = 3m$. . A further issue related to the scalar potential, which is an important check of all our calculation, is the possibility of computing the masses of the scalar fields by varying the linearized kinetic terms and the potential of (4.1), after power expansion of \mathcal{W} up to the second order in the scalar fields q_α^I . We find:

$$(\frac{\partial^2 \mathcal{W}}{\partial \sigma^2})_{\sigma=q=0, g=3m} = 48m^2 \quad (4.9)$$

$$(\frac{\partial^2 \mathcal{W}}{\partial q^{I0} \partial q^{J0}})_{\sigma=q=0, g=3m} = 8m^2 \delta^{IJ} \quad (4.10)$$

$$(\frac{\partial^2 \mathcal{W}}{\partial q^{Ir} \partial q^{Js}})_{\sigma=q=0, g=3m} = 24m^2 \delta^{IJ} \delta^{rs} \quad (4.11)$$

The linearized equations of motion become:

$$\square\sigma - 24m^2\sigma = 0 \quad (4.12)$$

$$\square q^{I0} - 16m^2 q^{I0} = 0 \quad (4.13)$$

$$\square q^{Ir} - 24m^2 q^{Ir} = 0 \quad (4.14)$$

If we use as mass unity the inverse AdS radius, which in our conventions is $R_{AdS}^{-2} = 4m^2$ we get:

$$\begin{aligned} m_\sigma^2 &= -6 \\ m_{q^{I0}}^2 &= -4 \\ m_{q^{Ir}}^2 &= -6 \end{aligned} \quad (4.15)$$

These values should be compared with the results obtained in reference [7] where the supergravity and matter multiplets of the AdS_6 $F(4)$ theory were constructed in terms of the singleton fields of the 5-dimensional conformal field theory, the singleton being given by hypermultiplets transforming in the fundamental of $\mathcal{G} \equiv E_7$. It is amusing to see that the values of the masses of the scalars computed in terms of the conformal dimensions are exactly the same as those given in equation (4.15).

This coincidence can be considered as a non trivial check of the AdS/CFT correspondence in six versus five dimensions.

To make contact with what follows we observe that the scalar squares masses in AdS_{d+1} are given by the $SO(2, d)$ quadratic Casimir [20]

$$m^2 = E_0(E_0 - d) \quad (4.16)$$

They are negative in the interval $\frac{d-2}{2} \leq E_0 < d$ (the lower bound corresponding to the unitarity bound i.e. the singleton) and attain the Breitenlohner-Freedman bound [15] when $E_0 = d - E_0$ i.e. at $E_0 = \frac{d}{2}$ for which $m^2 = -\frac{d^2}{4}$. Conformal propagation correspond to $m^2 = -\frac{d^2-1}{4}$ i.e. $E_0 = \frac{d\pm 1}{2}$. This is the case of the dilaton and triplet matter scalars.

5 $F(4) \otimes \mathcal{G}$ Superconformal Field Theory

Here we describe the basics of the $F(4)$ highest weight unitary irreducible representations “UIR’s” and exhibit two towers of short representations which are relevant for a K-K analysis of type IIA theory on (warped) $AdS_6 \otimes S^4$ [8],[9].

We will not consider here the \mathcal{G} representation properties but we will only concentrate on the supersymmetric structure.

Recalling that the even part of the $F(4)$ superalgebra is $SO(2, 5) \otimes SU(2)$, from a general result on Harish-Chandra modules [17],[11], [18], [19] of $SO(2, 2n+1)$ we know that there are only a spin 0 and a spin 1/2 singleton unitary irreducible representations [20], which, therefore, merge into a unique supersingleton representation of the $F(4)$ superalgebra: the hypermultiplet [7].

To describe shortening is useful to use a harmonic superfield language [12].

The harmonic space is in this case the 2-sphere³ $SU(2)/U(1)$, as in $N = 2$, $d = 4$ and

³The sphere is the simplest example of “flag manifold” whose geometric structure underlies the construction of harmonic superspaces [22]

$N = 1$, $d = 6$. A highest weight UIR of $SO(2, 5)$ is determined by E_0 and a UIR of $SO(5) \simeq Usp(4)$, with Dynkin labels (a_1, a_2) ⁴. We will denote such representations by $\mathcal{D}(E_0, a_1, a_2)$. The two singletons correspond to $E_0 = 3/2$, $a_1 = a_2 = 0$ and $E_0 = 2$, $a_1 = 1$, $a_2 = 0$.

In the AdS/CFT correspondence (E_0, a_1, a_2) become the conformal dimension and the Dynkin labels of $SO(1, 4) \simeq Usp(2, 2)$.

The highest weight UIR of the $F(4)$ superalgebra will be denoted by $\mathcal{D}(E_0, a_1, a_2; I)$ where I is the $SU(2)$ R -symmetry quantum number (integer or half integer).

We will show shortly that there are two (isolated) series of UIR's which correspond respectively to $1/2$ BPS short multiplets (analytic superfields) and intermediate short superfields. The former have the property that they form a ring under multiplication, as the chiral fields in $d = 4$ [21].

The first series is the massive tower of short vector multiplets whose lowest members is a massless vector multiplet in $Adj\mathcal{G}$ corresponding to the conserved currents of the \mathcal{G} global symmetry of the five dimensional conformal field theory.

The other series is the tower of massive graviton multiplets, which exhibit "intermediate shortening" and it is not of BPS type. Its lowest member is the supergravity multiplet which contains the $SU(2)$ R -symmetry current and the stress-tensor among the superfield components.

5.1 $F(4)$ superfields

The basic superfield is the supersingleton hypermultiplet $W^A(x, \theta)$, which satisfies the constraint

$$D_\alpha^{(A} W^{B)}(x, \theta) = 0 \quad (5.1)$$

corresponding to the irrep. $\mathcal{D}(E_0 = \frac{3}{2}, 0, 0; I = \frac{1}{2})$ [12].

By using harmonic superspace, (x, θ_I, u_i^I) , where $\theta_I = \theta_i u_I^i$, u_I^i is the coset representative of $SU(2)/U(1)$ and I is the charge $U(1)$ -label, from the covariant derivative algebra

$$\{D_\alpha^A, D_\beta^{jB}\} = i\epsilon^{AB}\partial_{\alpha\beta} \quad (5.2)$$

we have

$$\{D_\alpha^I, D_\beta^I\} = 0 \quad D_\alpha^I = D_\alpha^i u_i^I \quad (5.3)$$

Therefore from eq. (5.1) it follows the G -analytic constraint:

$$D_\alpha^1 W^1 = 0 \quad (5.4)$$

which implies

$$W^1(x, \theta) = \varphi^1 + \theta_2^\alpha \zeta_\alpha + d.t. \quad (5.5)$$

(d.t. means "derivative terms").

Note that W^1 also satisfies

$$D_\alpha^2 D^{2\alpha} W^1 = 0 \quad (5.6)$$

because there is no such scalar component⁵ in W^1 .

W^1 is a Grassman analytic superfield, which is also harmonic (that is $\mathbf{D}_2^1 W^1 = 0$ where,

⁴Note that the $Usp(4)$ Young labels h_1, h_2 are related to a_1, a_2 by $a_1 = 2h_2; a_2 = h_1 - h_2$.

⁵This is rather similar to the treatment of the $(1, 0)$ hypermultiplet in $D = 6$ [23]

using notations of reference [21], \mathbf{D}_2^1 is the step-up operator of the $SU(2)$ algebra acting on harmonic superspace).

Since W^1 satisfies $D^1 W^1 = 0$, any p -order polynomial

$$I_p(W^1) = (W^1)^p \quad (5.7)$$

will also have the same property, so these operators form a ring under multiplication [21], they are the $1/2$ BPS states of the $F(4)$ superalgebra and represent massive vector multiplets ($p > 2$), and massless bulk gauge fields for $p = 2$.

The above multiplets correspond to the $D(E_0 = 3I, 0, 0; I = \frac{p}{2})$ h.w. U.I.R.'s of the $F(4)$ superalgebra.

Note also that if W^1 carries a pseudo-real representation of the flavor group \mathcal{G} (e.g. **56** of $\mathcal{G} = E_7$) then W^1 satisfies a reality condition

$$(W^1)^* = W^2 \quad (5.8)$$

corresponding to the superfield constraint

$$(W^A)^{* \Lambda} = \epsilon_{AB} \Omega_{\Lambda\Sigma} W^{jB\Sigma} \quad (5.9)$$

The $SU(2)$ quantum numbers of the W^{1p} superfield components are:

$(\theta)^0$	$spin \ 0$	$I = \frac{p}{2}$
$(\theta)^1$	$spin \ \frac{1}{2}$	$I = \frac{p}{2} - \frac{1}{2}$
$(\theta)^2$	$spin \ 0 - spin \ 1$	$I = \frac{p}{2} - 1$
$(\theta)^3$	$spin \ \frac{1}{2}$	$I = \frac{p}{2} - \frac{3}{2}$
$(\theta)^4$	$spin \ 0$	$I = \frac{p}{2} - 2$

Note that the $(\theta)^4$ component is missing for $p = 2$, $p = 3$, while the $(\theta)^3$ component is missing for $p = 2$. However the total number of states is $8(p - 1)$ both for boson and fermion fields ($p \geq 2$).

The AdS squared mass for scalars is

$$m_s^2 = E_0(E_0 - 5) \quad (5.10)$$

so there are three families of scalar states with

$$\begin{aligned} m_1^2 &= \frac{3}{4}p(3p - 10) & p \geq 2 \\ m_2^2 &= \frac{1}{4}(3p + 2)(3p - 8) & p \geq 2 \\ m_3^2 &= \frac{1}{4}(3p + 4)(3p - 6) & p \geq 4 \end{aligned}$$

The only scalar states with $m^2 < 0$ are the scalar in the massless vector multiplet ($p = 2$) with $m_1^2 = -6$, $m_2^2 = -4$ (no states with $m^2 = 0$ exist) and in the $p = 3$ multiplet with

$$m^2 = -\frac{9}{4}.$$

We now consider the second "short" tower containing the graviton supermultiplet and its recurrences.

The graviton multiplet is given by $W^1\overline{W}^1$. Note that such superfield is not G -analytic, but it satisfies

$$D_\alpha^1 D^{1\alpha}(W^1\overline{W}^1) = D_\alpha^2 D^{2\alpha}(W^1\overline{W}^1) = 0 \quad (5.11)$$

this multiplet is the $F(4)$ supergravity multiplet. Its lowest component, corresponding to the dilaton in AdS_6 supergravity multiplet, is a scalar with $E_0 = 3$ ($m^2 = -6$) and $I = 0$. The tower is obtained as follows

$$G_{q+2}(W) = W^1\overline{W}^1(W^1)^q \quad (5.12)$$

where the massive graviton, described in eq.. (5.12) has $E_0 = 5 + \frac{3}{2}q$ and $I = \frac{q}{2}$. Note that the G_{q+2} polynomial, although not G -analytic, satisfies the constraint

$$D_\alpha^1 D^{1\alpha} G_{q+2}(W) = 0 \quad (5.13)$$

so that it corresponds to a short representation with quantized dimensions and highest weight given by $D(E_0 = 3 + 3I, 0, 0; I = \frac{q}{2})$.

We call these multiplets, following [23], "intermediate short" because, although they have some missing states, they are not BPS in the sense of supersymmetry. In fact they do not form a ring under multiplication.

It is worthwhile to mention that the towers given by (5.7), (5.12) correspond to the two isolated series of UIR's of the $F(4)$ superalgebra argued to exist in [11].

There are also long spin 2 multiplets containing 2^8 state where E_0 is not quantized and satisfies the bound $E_0 \geq 6$.

Finally let us make some comments on the role played by the flavour symmetry \mathcal{G} .

It is clear that, since the supersingleton W^1 is in a representation of \mathcal{G} (other than the gauge group of the world-volume theory), the I_p and G_{q+2} polynomials will appear in the p -fold and $(q+2)$ -fold tensor product representations of the \mathcal{G} group. This representation is in general reducible, however the 1/2 BPS states must have a first component totally symmetric in the $SU(2)$ indices and, therefore, only certain \mathcal{G} representations survive.

Moreover in the $(W^1)^2$ multiplet, corresponding to the massless \mathcal{G} - gauge vector multiplets in AdS_6 , we must pick up the adjoint representation $Adj\mathcal{G}$ and in $W^1\overline{W}^1$, corresponding to the graviton multiplet, we must pick up the \mathcal{G} singlet representation.

However in principle there can be representations in the higher symmetric and antisymmetric products, and the conformal field theory should tell us which products remains, since the flavor symmetry depends on the specific dynamical model.

The states discussed in this paper are expected to appear [8], [9] in the K-K analysis of IIA massive supergravity on warped $AdS_6 \otimes S^4$. It is amusing that superconformal field theory largely predicts the spectrum just from symmetry considerations. What is new in the $F(4)$ theory is the fact that, since it is not a theory with maximal symmetry, it allows in principle some rich dynamics and more classes of short representations than the usual compactification on spheres.

The K-K reduction is related to the horizon geometry of the $D4$ branes in a $D8$ brane background in presence of $D0$ branes [7], [8].

Conformal theories at fixed points of $5d$ gauge theories exist [10] which exhibit global symmetries $E_{N_f+1} \supset SO(2N_f) \otimes U(1)$, where $N_f \geq 1$ is the number of flavors (N_f $D8$ branes) and $U(1)$ is the “instantons charge” (dual to the $D0$ brane charge).

The E exceptional series therefore unifies perturbative and non perturbative series of the gauge theory.

It is natural to conjecture that a conformal fixed point $5d$ theory can be described by a singleton supermultiplet in the fundamental rep. of E_{N_f+1} . For the exceptional groups $N_f \geq 5$ these are the **27** of E_6 , the **56** of E_7 and the **248** of E_8 which are respectively complex, pseudo-real and real. The E_7 case was considered in ref. [7]. States coming from wrapped $D8$ branes will carry a non trivial representation of $SO(2N_f)$, which, together with some other states, must complete representations of E_{N_f+1} . It is possible that from the knowledge of $SO(2N_f)$ quantum numbers of supergravity in $D4-D8$ background one may infer the spectrum of E_{N_f+1} representations and then to realize these states in terms of boundary composite conformal operators. We hope to return to the above issues in a future investigation.

+

References

- [1] R. D’Auria, S.Ferrara and S.Vaulá, hep-th/0006107
- [2] L.J. Romans, Nucl. Phys. **B269** (1986) 691
- [3] M. Scheunert, W. Nahm and V. Rittenberg, J. Math. Phys. **17** (1976) 1640; W. Nahm, Nucl. Phys. **B135** (1978) 149
- [4] I. Bars and M. Günaydin, Comm. Math. Phys. **91** (1983) 31
- [5] Juan M. Maldacena, Adv.Theor.Math.Phys. **2** (1998) 231, Int.J.Theor.Phys. **38** (1999) 1113, hep-th/9711200; S. S. Gubser, I. R. Klebanov, A. M. Polyakov, Phys.Lett. **B428** (1998) 105, hep-th/9802109; E. Witten, Adv.Theor.Math.Phys. **2** (1998) 253, hep-th/9802150
- [6] O. Aharony, S. S. Gubser, J. Maldacena, H. Ooguri and Y. Oz, Phys.Rept. **323** (2000) 183, hep-th/9905111
- [7] S. Ferrara, A. Kehagias, H. Partouche, A. Zaffaroni, Phys.Lett. **B431** (1998) 57, hep-th/9804006
- [8] A. Brandhuber and Y. Oz, Phys.Lett. **B460** (1999) 307
- [9] M. Cvetič, H. Lü and C.N. Pope, Phys.Rev.Lett. **83** (1999) 5226, hep-th/9906221
- [10] N. Seiberg, Phys. Lett. **B388** (1966) 753, hep-th/960811; D. R. Morrison and N. Seiberg, Nucl. Phys. **B483** (1997) 229, hep-th/9609071; P. Intriligator, D. R. Morrison and N. Seiberg, Nucl. Phys. **B497** (1997) 56, hep-th/9702198
- [11] S. M. Minwalla, Theor. Math. Phys. **2** (1998) 781

- [12] A. Galperin, E. Ivanov, S. Kalitzin, V. Ogievetsky and E. Sokatchev, *Class. Quant. Grav.* **1** (1984) 469
- [13] L. Castellani, R. D'Auria and P. Fré, *Supergravity and Superstrings Vol 2* pag 794, World Scientific (1991)
- [14] A. Ceresole and G.G. Dall'Agata, hep-th/0004111
- [15] P. Breitenlohner and D. Z. Freedman, *Phys. Lett.* **B115** (1982) 197; *Ann. of Phys.* **144** (1982) 249; G. W. Gibbons, C. M. Hull and N. P. Warner, *Nucl. Phys.* **B218** (1983) 173; W. Boucher, *Nucl. Phys.* **B242** (1984) 282
- [16] S. Ferrara and L. Maiani, *Proc. V Silarg Symposium* (World Scientific, Singapore) (1986); S. Cecotti, L. Girardello and M. Porrati, *Nucl. Phys.* **B268**(1986) 295; A. Ceresole, R. D'Auria, S. Ferrara, P. Fré and E. Maina, *Phys. Lett.* **B268** (1986) 317
- [17] W. Siegel, *Int. Jou. Math. Phys.* **A4** (1989) 2015
- [18] E. Angelopoulos and M. Laoues, *Rev. Math. Phys.* **10** (1998) 271, hep-th/9806100
- [19] S. Ferrara, C. Fronsdal, hep-th/0006009
- [20] M. Flato and G. Fronsdal, *Lett. Math. Phys.* **2** (1978) 421; *Phys. Lett.* **97B** (1980) 236; *J. Math. Phys.* **22** (1981) 1100; *Phys. Lett.* **B172** (1986) 412
- [21] S. Ferrara and E. Sokatchev, hep-th/000515
- [22] G. G. Hartwell and P. S. Howe, *Class. Quant. Grav.* **12** (1995) 1823; *Int. J. Mod. Phys.* **10** (1995) 3901
- [23] S. Ferrara and E. Sokatchev, hep-th/0001178